MATHEMATICAL METHODS IN ECONOMICS – II

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BY

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B.A. 2ND SEMESTER STUDY MATERIALS /CLASS NOTES

2nd SEMESTER

B.A……………………….(Ritu Madam 20/04/2020)

Equlibrium of Multiplant Firm

When a firm produces a product (homogeneous) in more than one plant under different cost behaviour then the profit maximizing condition of the firm differs from the normal condit

Suppose a firm produces output in

<div style="display:flex; justify-content:space-between;">

PLANT 1   PLANT 2

Q_1 Q_2

TC_1 TC_2

$TC_1 = C_1(Q_1)$ $TC_2 = C_2(Q_2)$

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And $Q = Q_1 + Q_2$ (i.e =total output is equal to output in PLANT1 & PLANT2)

Hence $\prod = TR - TC$

$\prod = TR - (TC_1 + TC_2) = TR - TC_1 - TC_2$

Maximization of profit

1st order condition

$\partial\prod /\partial Q_1 = 0$ and $\partial\prod / \partial Q_2 = 0$

2nd order condition

$\left| H_1 \right| < 0$ and $\left| H_2 \right| > 0$

Where $\left| H_2 \right| = \begin{vmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{vmatrix}$

Solution to last class example

: Q_1 = 4.14 Q_2 = 6.76 Q_3 = 4.82

Profit = 469.79

AR_1 = 33.72

AR_2 = 45.72

AR_3 = 44.72

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Solution to multi plant monopolist

Q_1 = 6 Q_2 = 4 and Profit = 243

Example : A monopolist discriminates prices between two markets 1 and 2 and his average functions are given by

   $AR_1 = P_1 = 55 - 4 Q_1$

   $AR_2 = P_2 = 25 - 3 Q_2$

The TC = $20 - 5 Q + Q^2$

(i) Find profit maximzing   output to be sold in the two markets

(ii) Show that the market with higher elasticity of demand has lower price and vice versa

Example : A monopolist produces his output in three plants and his total output is Q = $Q_1 + Q_2 + Q_3$
His total cost function in each of the plants are

$TC_1 = 10 - 2 Q_1 + Q_1^2$

$TC_2 = 20 - 2 Q_2 + 1/2 Q_2^2$

$TC_3 = 50 - 12Q_3 + 2Q_3^2$ and his

AR = 60 - 5 Q

Find (i) output to be produced in each of the plants

(ii) maximum profit and equilibrium price

Equilibrium of a Multi Product Firm

A firm produces more than one product. In such situation total revenue of the firm will depend on more than one variable. We will discuss the equilibrium of a firm producing two commodities, Q_1 and Q_2, jointly (a) under the condition of perfect competition as well as (b) under imperfect condition, say monopoly.

When a firm produces two products Q_1 and Q_2 jointly, the total cost will be a function of Q_1 and Q_2 and so we define

$TC = C(Q_1, Q_2)$

with $MC_1 = \partial C/\partial Q_1 = C_1(Q_1, Q_2)$ representing the marginal cost of first product and

$MC_2 = \partial C/\partial Q_2 = C_2(Q_1, Q_2)$ as the marginal cost of second product assuming that the marginal cost of both products are individually functions of Q_1 and Q_2.

In a perfectly competitive market, the prices of Q_1 and Q_2 are given to the firm.

If prices are denoted by P_1 and P_2 respectively, the total revenue function will be

$TR = P_1 * Q_1 + P_2 * Q_1$

such that

$MR_1 = \partial(TR)/\partial Q_1 = P_1$ and $MR_2 = \partial(TR)/\partial Q_2 = P_2$

representing marginal revenues of first and second products respectively.

Thus the profit function of multi-product firm can be expressed as

$\prod = TR - TC = P_1 * Q_1 + P_2 * Q_1 - C(Q_1, Q_2)$.

Example : Refer Srinath Barua

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